Dilaton stabilization and baryogenesis

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Entropy production by dilaton decay is studied in the model where the dilaton acquires a potential via gaugino condensation in the hidden gauge group. Its effect on Affleck-Dine baryogenesis is investigated with and without nonrenormalizable terms in the potential. It is shown that the baryon asymmetry produced by this mechanism with higher-dimensional terms is diluted by the dilaton decay and can be regulated to the observed value.

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I. INTRODUCTION

Many gauge-singlet scalar fields arise in effective four-dimensional supergravity that could be derived from string theories. Among them, the dilaton S has a flat potential in all orders in perturbation theory [1]. Therefore some nonperturbative effects are expected to generate a potential whose minimum corresponds to the vacuum expectation value (VEV). The most promising mechanism of dilaton stabilization and supersymmetry breaking is gaugino condensation in the hidden gauge sector [2–5].

In cosmological considerations, even if the dilaton acquires a potential through such nonperturbative effects, there are some difficulties in relaxing the dilaton to the correct minimum, as pointed out by Brustein and Steinhardt [6]. Since the potential generated by multiple gaugino condensations is very steep in the small field-value region, the dilaton would have large kinetic energy and overshoot the potential maximum to run away to infinity.

As a possible way to overcome this problem, Barreiro *et al.* [7] pointed out that the dilaton could slowly roll down to its minimum with a little kinetic energy due to the large Hubble friction if the background fluid dominated the cosmic energy density. As a result, the dilaton could be trapped and would oscillate around the minimum.

When the dilaton decays, however, the remaining energy density is transformed to radiation. One may worry about a huge entropy production by the decay, because it could dilute the initial baryon asymmetry [8]. Therefore, in order to obtain the observed baryon asymmetry, as required by, e.g., nucleosynthesis, it is necessary to produce a larger asymmetry than the one observed at the outset.

An attractive mechanism to produce large baryon asymmetry in supersymmetric models was proposed by Affleck and Dine [9]. However, the baryon-to-entropy ratio produced by this mechanism, n_b/s, is usually too large. So if we take into account the entropy production after the baryogenesis, we can expect that the additional entropy may dilute the excessive baryon asymmetry to the observed value, as pointed out in, e.g., Refs. [10,11].

In this paper, we investigate whether dilution by dilaton decay can regulate the large baryon asymmetry produced by the Affleck-Dine mechanism to the observed value. The paper is organized as follows. In Secs. II and III, we describe the potential and the dynamics of the dilaton. Then, in Sec. IV we estimate the baryon asymmetry generated by the Affleck-Dine mechanism, taking into account dilution by dilaton decay. Section V is devoted to the conclusion. We take units with $8\pi G=1$.

II. DILATON POTENTIAL

We consider the potential of the dilaton nonperturbatively induced by multiple gaugino condensates. In string models, the tree level Kähler potential is given by

$$K=-\ln(S+S^*)-3\ln(T+T^*-|\Phi|^2), \tag{1}$$

where S is the dilaton, T is the modulus, and Φ represents some chiral matter fields [12].

The effective superpotential of the dilaton [2] generated by gaugino condensation is given by

$$W=\sum_a \Lambda_a(T)e^{-\alpha_a S}. \tag{2}$$

Here, for the $SU(N_a)$ gauge group and the chiral matter in $M_a(N_a+\overline{N}_a)$ "quark" representations, α_a and Λ_a are given by

$$\alpha_a=\frac{8\pi^2}{N_a-M_a/3}, \tag{3}$$

$$\Lambda_a=-\frac{N_a-M_a/3}{\eta^6(T)}(32\pi^2 e)^{3(N_a-M_a/3)/(3N_a-M_a)} \times\left(\frac{M_a}{3}\right)^{M_a/(3N_a-M_a)}. \tag{4}$$

Then the modulus T has a potential minimum due to the presence of the Dedekind function $\eta(T)$ [5]. Hereafter, we assume the stabilization of the modulus T at the potential minimum and concentrate on the evolution of the real part of the dilaton field.

Since at least two condensates are required to form the potential minimum, we consider a model with two condensates. Then the indices of the gauge group are $a=1,2$, and we take $10\lesssim\alpha_1\lesssim\alpha_2$.

The potential V for scalar components in supergravity is given by

$$V=e^K[(K^{-1})^i{}_j D_i W(D_j W)^*-3|W|^2], \tag{5}$$

where

$$D_i W=\frac{\partial W}{\partial\Phi^i}+\frac{\partial K}{\partial\Phi^i}W, \tag{6}$$

$K_i{}^j=\partial^2 K/\partial\Phi^i\partial\Phi_j^*$, the inverse $(K^{-1})^i{}_j$ is defined by $(K^{-1})^i{}_j K^j{}_k=\delta^i{}_k$, and $i=S,T,\Phi$. In the region $\alpha\mathrm{Re}S\gg 1$, the potential Eq. (5) can be rewritten as

$$\begin{aligned}V(S)&\simeq e^K(K^{-1})^S{}_S|\partial_S W|^2\\ &=(S+S^*)|\alpha_1\Lambda_1|^2 e^{-\alpha_1(S+S^*)}\\ &\quad\times\left|1+\frac{\alpha_2\Lambda_2}{\alpha_1\Lambda 1}e^{-(\alpha_2-\alpha_1)S}\right|^2.\end{aligned} \tag{7}$$

In this potential the imaginary part of S has a minimum at

$$\mathrm{Im}S_{\mathrm{min}}=\frac{(2n+1)\pi}{\alpha_1-\alpha_2}, \tag{8}$$

where n is an integer. So we assume $\mathrm{Im}S=\mathrm{Im}S_{\mathrm{min}}$ and concentrate on the behavior of ReS hereafter. Then we find that the potential minimum $\mathrm{Re}S_{\mathrm{min}}$ is given by

$$\mathrm{Re}S_{\mathrm{min}}=\frac{1}{\alpha_2-\alpha_1}\ln\left(\frac{\alpha_2}{\alpha_1}\frac{\Lambda_2}{\Lambda_1}\right). \tag{9}$$

We assume $\mathrm{Re}S_{\mathrm{min}}\simeq 2$ to reproduce a phenomenologically viable value of the gauge coupling constant of the grand unified theory [5]. The negative vacuum energy at the minimum of the potential is assumed to be canceled by some mechanism such as a vacuum expectation value of three-form field strength. Although we consider a model with two gaugino condensates, our following estimations would be almost unchanged in a single condensate model with nonperturbative Kähler corrections [13,14], because the evolution of the dilaton is determined only by the slope of the potential in the region $S\ll S_{\mathrm{min}}$ and its mass.

On the other hand, the position of the local maximum of the potential, $\mathrm{Re}S_{\mathrm{max}}$, is given by

$$\mathrm{Re}S_{\mathrm{max}}=\mathrm{Re}S_{\mathrm{min}}+\frac{1}{\alpha_2-\alpha_1}\ln\left(\frac{\alpha_2}{\alpha_1}\right). \tag{10}$$

For $S\ll S_{\mathrm{min}}$, the potential (7) can be approximated as

$$V(S)\simeq 2\mathrm{Re}S V_0 e^{-\alpha_2 2\mathrm{Re}S}, \tag{11}$$

where $V_0\simeq|\alpha_2\Lambda_2|^2$. The potential (7) has a minimum at S_{min} and a local maximum at $S_{\mathrm{max}}(>S_{\mathrm{min}})$. For $S\gtrsim S_{\mathrm{cr}}\equiv S_{\mathrm{min}}-1/(\alpha_2-\alpha_1)$, the approximate expression for the potential (11) breaks down. Around the potential minimum S_{min}, the potential (7) becomes

$$V(S)\simeq|\alpha_1\Lambda_1|^2 e^{-2\alpha_1 S_{\mathrm{min}}}(\alpha_2-\alpha_1)^2(\mathrm{Re}S-\mathrm{Re}S_{\mathrm{min}})^2. \tag{12}$$

However, one can see from the Kähler potential (1) that the variable Re S does not have a canonical kinetic term. Therefore we introduce the canonically normalized variable ϕ as

$$\phi\equiv\frac{1}{\sqrt{2}}\ln\mathrm{Re}\,S. \tag{13}$$

III. STABILIZATION MECHANISM FOR THE DILATON

Here, after reviewing the mechanism for dilaton stabilization proposed by Barreiro *et al.* [7], we estimate the relic energy density of the dilaton and the amount of entropy density produced by its decay. We will consider the situation that the universe after inflation contains the dilaton ϕ and a fluid with the equation of state $p=(\gamma-1)\rho$, where γ is a constant. For example, $\gamma=4/3$ for radiation or $\gamma=1$ for nonrelativistic matter. The latter includes an oscillating inflaton field or/and the Affleck-Dine (AD) condensate ϕ_{AD}.

In the spatially flat Robertson-Walker space-time,

$$ds^2=-dt^2+a(t)^2 d\boldsymbol{x}^2, \tag{14}$$

with the scale factor $a(t)$, the Friedmann equations and the field equation for ϕ read

$$\dot{H}=-\frac{1}{2}(\rho+p+\dot{\phi}^2), \tag{15}$$

$$\ddot{\phi}=-3H\dot{\phi}-\frac{dV(\phi)}{d\phi}, \tag{16}$$

$$H^2=\frac{1}{3}\left[\rho+\frac{1}{2}\dot{\phi}^2+V(\phi)\right], \tag{17}$$

where $H=\dot{a}/a$ is the Hubble parameter and the overdot denotes time differentiation. We define the new variables

$$x\equiv\frac{\dot{\phi}}{\sqrt{6}H},\quad y\equiv\frac{\sqrt{V(\phi)}}{\sqrt{3}H},\tag{18}$$

and the number of e-folds $N\equiv\ln(a)$.

Then the equations of motion can be rewritten as

$$x'=-3x-\sqrt{\frac{3}{2}}\frac{\partial_\phi V}{V}y^2+\frac{3}{2}x[2x^2+\gamma(1-x^2-y^2)],\tag{19}$$

$$y'=\sqrt{\frac{3}{2}}\frac{\partial_\phi V}{V}xy+\frac{3}{2}y[2x^2+\gamma(1-x^2-y^2)],\tag{20}$$

$$H'=-\frac{3}{2}H[2x^2+\gamma(1-x^2-y^2)],\tag{21}$$

where the prime denotes a derivative with respect to N. In terms of these variables, the Friedmann equation becomes $x^2+y^2+\rho/(3H^2)=1$. We see that x^2 and y^2 are, respectively, the ratios of the kinetic and potential energy densities of the dilaton to the total energy density. We consider the case of the universe dominated by the background fluid, so the inequalities $x^2,y^2\ll 1$ hold. Then Eq. (21) can easily be solved and the solution is

$$H=H_0e^{-3\gamma N/2}.\tag{22}$$

Next we introduce another new variable,

$$x_s\equiv\frac{d\,\mathrm{Re}\,S}{d\phi}x.\tag{23}$$

Then Eqs. (19) and (20) can be rewritten, respectively, as

$$x_s'=-3x_s+\frac{3}{2}\gamma x_s+2\alpha_2\sqrt{\frac{3}{2}}\left(\frac{d\,\mathrm{Re}\,S}{d\phi}\right)^2y^2,\tag{24}$$

$$y'=-2\alpha_2\sqrt{\frac{3}{2}}x_sy+\frac{3}{2}\gamma y,\tag{25}$$

where we have used the relation $-2\alpha_1V=\partial V/\partial\,\mathrm{Re}\,S$. Now we examine the stationary points in the above equations. From $y'=0$, we find

$$x_s=\sqrt{\frac{3}{2}}\frac{\gamma}{2\alpha_2}.\tag{26}$$

On the other hand, from $x_s'=0$ we see that

$$y^2=\frac{3(2-\gamma)\gamma}{2(2\alpha_2)^2}\left(\frac{d\,\mathrm{Re}\,S}{d\phi}\right)^{-2}.\tag{27}$$

Except for the factor $(d\,\mathrm{Re}\,S/d\phi)^{-2}$, Eqs. (26) and (27) represent the scaling solution for a scalar field with exponential potential [15]. In spite of the presence of the factor $(d\,\mathrm{Re}\,S/d\phi)^{-2}$, we can verify that the deviation from the scaling solution is small enough, as already shown in [7]. Thus we can write the solution as

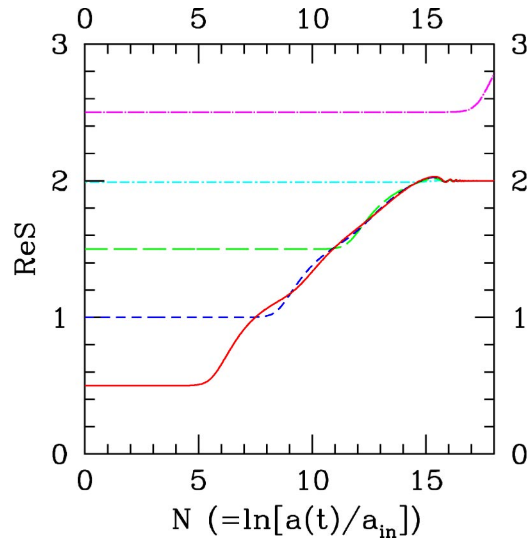

FIG. 1. Evolution of $\mathrm{Re}\,S$ as a function of N for various initial conditions with $S_{\mathrm{min}}=2$ in the case of $\gamma=4/3$. We set the initial values of the velocity and the Hubble expansion rate as $d\,\mathrm{Re}\,S/dt|_0=0$ and $H_0=1$, respectively.

$$\mathrm{Re}\,S=\frac{3\gamma}{2\alpha_2}N+\frac{1}{\alpha_2}\ln\left[\frac{4V_0}{(2-\gamma)\gamma}\left(\frac{2\alpha_2}{3H_0}\right)^2\right]+\epsilon(N),\tag{28}$$

where

$$\epsilon(N)=\frac{2}{\alpha_2}\ln\left\{\frac{3\gamma}{2\alpha_2}N+\frac{1}{\alpha_2}\ln\left[\frac{4V_0}{(2-\gamma)\gamma}\left(\frac{2\alpha_2}{3H_0}\right)^2\right]\right\}\tag{29}$$

denotes the deviation from the scaling solution; we find $x_s'=\epsilon(N)''\ll 1$.

Figure 1 depicts the time evolution of $\mathrm{Re}\,S$ as a function of N for various initial values of the dilaton field amplitude. As is seen there, if the dilaton relaxes to the scaling solution before reaching S_{min}, its energy is small enough to prevent overshooting. Attractor behavior in a different situation has been studied in [16]. The Hubble parameter at $S=S_{\mathrm{min}}$ is estimated as

$$H_{\mathrm{min}}\simeq\frac{2\alpha_2}{3}\sqrt{\frac{2V_0}{(2-\gamma)\gamma}}e^{-\alpha_2S_{\mathrm{min}}}\equiv t_{\mathrm{min}}^{-1}\tag{30}$$

from Eqs. (22) and (28). On the other hand, as is seen from Eq. (7), the mass of the dilaton in vacuum is given by

$$m_\phi\simeq 2(\alpha_2-\alpha_1)\sqrt{V_0}e^{-\alpha_2S_{\mathrm{min}}}.\tag{31}$$

We therefore find $m_\phi\simeq H_{\mathrm{min}}$, so the dilaton begins to oscillate immediately when it approaches the potential minimum. Since the mass of the gravitino is given by $m_{3/2}\simeq\Lambda_2e^{-\alpha_2S_{\mathrm{min}}}$, the mass of the dilaton is

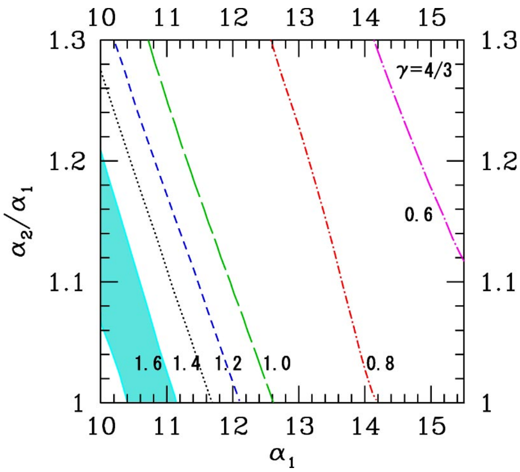

FIG. 2. Energy density of the dilaton at the beginning of the oscillations for various model parameters. The number associated with each contour line represents the value of ρ_ϕ normalized by $10^{-4}\rho$. Here we adopt $\gamma=4/3$.

$$m_\phi \simeq \alpha_1^2 m_{3/2} \simeq (10^2\ \text{TeV})\left(\frac{m_{3/2}}{1\ \text{TeV}}\right)\left(\frac{\alpha_1}{10}\right)^2. \qquad (32)$$

When the dilaton S approaches the critical point $S_{\rm cr}$, the single exponential approximation (11) breaks down and the scaling behavior terminates. Then the energy density of the dilaton is estimated as

$$\begin{aligned}\rho_\phi^{in} &= \frac{1}{2}\dot{\phi}^2 + V\bigg|_{t_{\min}} = 3H^2(x^2+y^2)\bigg|_{t_{\min}} \\ &= \frac{3}{2}\left(\frac{\gamma}{2\alpha_1}\right)^2 \rho \left(\frac{d\,\text{Re}\,S}{d\phi}\right)^{-2}\left(1+\frac{2-\gamma}{2\gamma}\right)\bigg|_{t_{\min}} \\ &\simeq 10^{-3}\rho\left(\frac{10}{\alpha_1}\right)^2\left(\frac{2}{\text{Re}\,S}\right)^2 \quad \text{for} \quad \gamma=4/3. \qquad (33)\end{aligned}$$

Indeed, the numerical calculation gives a close value $\rho_\phi \simeq 10^{-4}\rho$ at the beginning of the oscillation. After that, the dilaton begins to oscillate and the energy density decreases as $a(t)^{-3}$ until it decays.

In Fig. 2, we present the energy density of the dilaton in a universe filled by the $\gamma=4/3$ background fluid at the beginning of the oscillation regime for various model parameters α_1 and α_2. The values along the contour lines represent the energy density ρ_ϕ in units of $10^{-4}\rho$. The case with $\gamma=1$ is depicted in Fig. 3, where we find a smaller energy density of the dilaton by a factor of ~ 3. These figures are drawn in the two-parameter space, although the potential contains four parameters as seen from Eq. (7). The other two have been fixed by setting $m_{3/2}=1$ TeV and $S_{\min}=2$ [5].

The decay of the dilaton produces huge entropy. If we assume that the background fluid is radiation with $\gamma=4/3$, at the time $t=t_{\min}$ its temperature is $T\sim 10^{11}$ GeV and the entropy density is

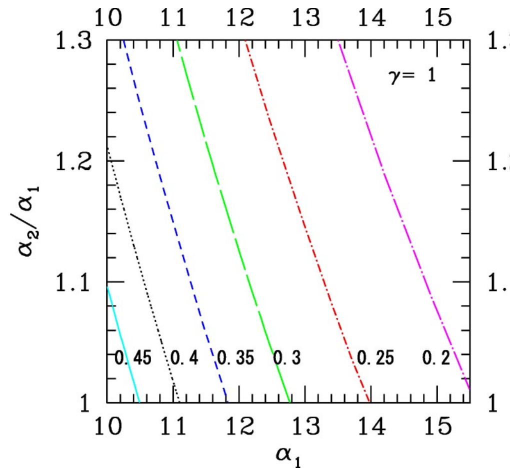

FIG. 3. Same as Fig. 2 except for $\gamma=1$.

$$s = \frac{4\pi^2}{90} g_* T^3, \qquad (34)$$

where $g_* \sim 10^2$ is the effective number of relativistic degrees of freedom. By using Eq. (33), we find that the entropy density increases by the factor

$$\Delta = \frac{T}{T_D}\frac{\rho_\phi}{\rho}\bigg|_{t_{\min}} \simeq 10^9 \left(\frac{T}{10^{11}\ \text{GeV}}\right)\left(\frac{10^{-2}\ \text{GeV}}{T_D}\right) \times \left(\frac{\rho_\phi/\rho|_{H_{\min}^{-1}}}{10^{-4}}\right) \qquad (35)$$

to the moment of the dilaton decay when

$$H = \Gamma_D \simeq m_\phi^3 \simeq (10^{-21}\ \text{GeV})\left(\frac{m_\phi}{10^2\ \text{TeV}}\right)^3, \qquad (36)$$

where

$$T_D \simeq m_\phi^{3/2} \simeq (10^{-1}\ \text{GeV})\left(\frac{m_\phi}{10^2\ \text{TeV}}\right)^{3/2} \qquad (37)$$

is the reheating temperature after decay of the dilaton.

IV. AFFLECK-DINE BARYOGENESIS

The Affleck-Dine mechanism is an efficient mechanism of baryogenesis in supersymmetric models [9,17]. In fact, it is too efficient and the baryon asymmetry produced, n_b/s, is in general too large. However, additional entropy release by the dilaton decay may significantly dilute the baryon asymmetry [10,11], and we examine this possibility here.

It is known that Q-ball formation occurs for many Affleck-Dine flat directions [17–19]. Whether Q balls form or not depends on the shape of the radiative correction to the flat direction [19,20]. In this paper we estimate the baryon asymmetry providing that the Affleck-Dine field does not lead to Q-ball formation. For instance, one example is a flat direction with large mixtures of top squarks in the case of light gaugino masses; another is the $H_u L$ direction [20].

A. Original Affleck-Dine mechanism

First, we investigate the originally proposed Affleck-Dine mechanism with a flat potential up to $\phi_{AD}\sim 1$ [9]. We consider the situation that there are the dilaton and the Affleck-Dine condensate in a radiation dominated universe. As mentioned above, at the moment $H=m_\phi$, the dilaton begins to oscillate with initial energy density $\rho_\phi \simeq 10^{-4}\rho_\gamma$, where ρ_γ is the energy density of the background radiation. Then, on the other hand, the AD condensate is expected to take a large expectation value, $\phi_{AD}\sim 1$, above which its potential blows up exponentially.

The amplitude of the AD field and its energy density remain constant while the Hubble parameter is larger than m_{AD}, where m_{AD} is the mass of the AD condensate. One should keep in mind, however, that although the energy density ρ_{AD} remains constant the baryon number density decreases as $1/a^3$ if baryonic charge is conserved. So if baryon charge is accumulated in ''kinetic'' motion of the phase of ϕ_{AD} it will decrease as $1/a^3$. If, on the other hand, the AD field is frozen at the slope of a non-spherically-symmetric potential then the baryonic charge of the AD field is not conserved and after $H<m_{AD}$ both radial and angular degrees of freedom will be ''defrosted'' and the baryonic charge may be large.

As we noted above, when $H\simeq m_{3/2}$ $(\simeq m_{AD})$, the AD field begins to oscillate. Its energy density at that moment becomes comparable to that of the radiation, while the energy density of the dilaton is estimated as

$$\rho_\phi|_{H=m_{3/2}} = \left(\frac{m_\phi}{m_{3/2}}\right)^{1/2} \frac{\rho_\phi}{\rho_\gamma}\bigg|_{H=m_\phi} \rho_\gamma|_{H=m_{3/2}} \simeq 10^{-3}\rho_\gamma|_{H=m_{3/2}} \tag{38}$$

for the initial energy density $\rho_\phi = 10^{-4}\rho_\gamma$ and $m_\phi \simeq 10^2 m_{AD}$. After that the universe becomes dominated by the oscillating AD condensate and enters into the approximately matter dominated regime (see below).

The energy density of the condensate and its baryon number density are given, respectively, by the expressions

$$\rho_{AD}=m_{AD}^2\phi_{AD}^2, \quad n_b=\kappa m_{AD}\phi_{AD}^2, \tag{39}$$

where $\kappa = n_b/n_{AD}<1$ is a numerical coefficient and n_{AD} is the number density of the AD field.

The rate of evaporation of the condensate, given by the decay width of the AD field into fermions, $\Gamma_{AD}=Cm_{AD}$ with $C=0.1-0.01$, is quite large. When the Hubble parameter becomes smaller than Γ_{AD}, thermal equilibrium will be established rather soon. However, the condensate will evaporate very slowly and disappear much later [21]. The low evaporation rate is related to the large baryonic charge and relatively small energy density of the condensate. Below we will find the temperature and the moment of the condensate evaporation, repeating the arguments of Ref. [21]. Let us assume that the condensate evaporated immediately when $H=\Gamma_{AD}$, producing a plasma of relativistic particles with temperature T_{AD} and chemical potential μ_{AD}. The temperature can be estimated as $T_{AD}\simeq \rho_{AD}^{1/4}$ and since $T_{AD}\gg m_{AD}$ the chemical potential is given by

$$\mu_{AD}\simeq \frac{n_b}{T_{AD}^2}=\kappa\phi_{AD}\gg m_{AD}, \tag{40}$$

if κ is not very small. On the other hand, the chemical potential of the bosons cannot exceed their mass. This means that instantaneous evaporation of the condensate is impossible. The process of evaporation proceeds rather slowly with an almost constant temperature of the created relativistic plasma. During the process of evaporation the energy density of the latter was small in comparison with the energy density of the condensate, except for the final stage when the condensate disappeared.

The cosmological baryon number density and energy densities are given by the equilibrium expressions

$$\frac{n_{b,\mathrm{tot}}}{T^3} = \frac{2N_fN_cB_q}{6\pi^2}(\xi_q^3+\pi^2\xi_q) + \frac{1}{2\pi^2}\int_0^\infty d\eta\,\eta^2\left[\frac{1}{\exp(\epsilon-\xi)-1} - \frac{1}{\exp(\epsilon+\xi)-1}\right] + B_c \tag{41}$$

$$\frac{\rho_{\mathrm{tot}}}{T^4} = \frac{\pi^2 g_*}{30} + \frac{7}{8}\,\frac{2N_f(N_c+1)\pi^2}{15}\left[1+\frac{30}{7}\left(\frac{\xi_q}{\pi}\right)^2+\frac{15}{7}\left(\frac{\xi_q}{\pi}\right)^4\right] + \frac{1}{2\pi^2}\int_0^\infty d\eta\,\eta^2\epsilon\left[\frac{1}{\exp(\epsilon-\xi)-1}+\frac{1}{\exp(\epsilon+\xi)-1}\right] + \rho_c, \tag{42}$$

where $\eta\equiv p/T$ is the dimensionless momentum, $\epsilon \equiv \sqrt{\eta^2+m_{AD}^2/T^2}$, and $\xi\equiv\mu_{AD}/T$ and $\xi_q\equiv\mu_q/T$ are the dimensionless chemical potentials of the AD field and the quarks, respectively. $N_f=6$ and $N_c=3$ are the numbers of flavors and colors and the factor 2 came from counting spin states, $B_q=1/3$ is the baryonic charge of quarks while the baryonic charge of the AD field is assumed to be 1, and B_c and ρ_c are the baryon number density and energy density of the condensate normalized to T^3 and T^4, respectively. The first term in ρ_{tot} includes the energy density of light particles with zero charge asymmetry and g_* is the number of their species. The second term includes the contribution from leptons with the same chemical potential as the quarks—it is given by (N_c+1).

For definiteness, let us assume that the AD field decays into the channel $\phi_{AD}\to 3q+l$ and, taking into account that the sum of baryonic and leptonic charges is conserved,[1] so that $B-L=0$, we find

$$\mu_q=\mu_l=\frac{\mu_{AD}}{4}. \tag{43}$$

Before complete evaporation of the condensate, the chemical potential of the AD field remains constant and equal to its maximum allowed value m_{AD}. Thus the only unknowns in these expressions are the temperature and the amplitude of the field in the condensate. According to Eq. (39), from Eqs. (41) and (42) we obtain

$$\frac{B_c}{\rho_c}=\kappa\frac{T}{m_{AD}}. \tag{44}$$

The same relation was true for the initial values of $n_{b,\mathrm{tot}}/T^3$ and ρ_{tot}/T^4. Assuming that the ratio $n_{b,\mathrm{tot}}/\rho_{\mathrm{tot}}$ remains the same during almost the whole process of evaporation, although it is not exactly so, we can exclude B_c, $n_{b,\mathrm{tot}}$, ρ_c, and ρ_{tot} from expressions (41) and (42) and find one equation that permits us to calculate the plasma temperature in the presence of an evaporating condensate as a function of the baryonic charge fraction in the initial condensate, κ. We find

$$m_{AD}/T\simeq\begin{cases}20 \quad \text{for} \quad \kappa=1, & (45)\\ 2 \quad \text{for} \quad \kappa=0.1. & (46)\end{cases}$$

An exact solution of the problem demands a much more complicated study of the evolution of the energy density according to the equation $\dot{\rho}=-3H(\rho+P)$, while the evolution of the baryonic charge density is determined by the conservation of baryonic charge, which is assumed to be true at the stage under consideration, and thus $n_{b,\mathrm{tot}}\propto a^{-3}$. The temperature of plasma found in this way would not be much different from the approximate expressions presented above.

Using the above-calculated plasma temperature (45) we find that at the moment of condensate evaporation (when $\mu_{AD}=m_{AD}$) the cosmological energy and baryon number densities of the created relativistic plasma are given by

$$\rho_p\simeq 1000T^4, \tag{47}$$

$$n_b\simeq 50T^3 \tag{48}$$

for $\kappa=1$, and

$$\rho_p\simeq 70T^4, \tag{49}$$

$$n_b\simeq 1.75T^3 \tag{50}$$

for $\kappa=0.1$.

We see that a large baryon asymmetry prevents fast condensate evaporation, although the interaction rate could be much larger than the expansion rate. From Eq. (48) or Eq. (50) and the baryon number conservation

$$n_b=\kappa m_{AD}\left(\frac{a_{AD}}{a(t)}\right)^3(\phi_{AD}|_{H=m_{AD}})^2, \tag{51}$$

we find

$$\left(\frac{a_{\mathrm{ev}}}{a_{AD}}\right)^3=\kappa\frac{m_{AD}}{n_b}(\phi_{AD}|_{H=m_{AD}})^2 \tag{52}$$

$$\simeq\begin{cases}160\left(\dfrac{\phi_{AD}|_{H=m_{AD}}}{m_{AD}}\right)^2\simeq 10^{33} \quad \text{for} \quad \kappa=1, & (53)\\ 0.46\left(\dfrac{\phi_{AD}|_{H=m_{AD}}}{m_{AD}}\right)^2\simeq 3\times 10^{30} \quad \text{for} \quad \kappa=0.1, & (54)\end{cases}$$

where a_{AD} and a_{ev} are the value of the scale factor at the moment $H=m_{AD}$ and that at the evaporation of the AD field, respectively. Then at the evaporation the Hubble parameter and the baryon-to-entropy ratio are respectively given by

$$H_{\mathrm{ev}}=(\rho_p/3)^{1/2}\simeq\begin{cases}2\times 10^{-14}\ \text{GeV} \quad \text{for} \quad \kappa=1,\\ 5\times 10^{-13}\ \text{GeV} \quad \text{for} \quad \kappa=0.1,\end{cases} \tag{55}$$

$$\left.\frac{n_b}{s}\right|_{\mathrm{ev}}\simeq\begin{cases}1 \quad \text{for} \quad \kappa=1, & (56)\\ 0.04 \quad \text{for} \quad \kappa=0.1, & (57)\end{cases}$$

from Eqs. (47), (48), (49), and (50).

Now we have to calculate the ratio of the baryon asymmetry to the entropy of the plasma after thermalization of the products of dilaton decay. Initially, at the moment of evaporation of the AD condensate the energy density of the dilaton is roughly 10^{-3} with respect to the energy density of the plasma. The latter is dominated by the chemical potential $\mu=m_{AD}>T$. When the universe expands by the factor $\rho_{AD}/\rho_\phi|_{\mathrm{ev}}\equiv a_{\mathrm{eq}}/a_{\mathrm{ev}}\simeq 10^3$ the dilaton starts to dominate and the relativistic expansion regime turns into the matter dominated one at $a=a_{\mathrm{eq}}$. At the moment of dilaton decay, the energy density of the dilaton becomes larger than the energy density of the plasma formed by the evaporation of the AD condensate by the factor $a_{\mathrm{d}}/a_{\mathrm{eq}}=(H_{\mathrm{eq}}/H_{\mathrm{d}})^{2/3}$, where a_{d} and H_{d} are the scale factor and the Hubble parameter at the time of the dilaton decay. Keeping in mind that $H_{\mathrm{eq}}=(a_{\mathrm{eq}}/a_{\mathrm{ev}})^2H_{\mathrm{ev}}=10^{-6}H_{\mathrm{ev}}$, we obtain the dilution factor by dilaton decay as

$$\Delta=\left.\left(\frac{\rho_\phi}{\rho_{AD}}\right)^{3/4}\right|_{\mathrm{d}}=\left(\frac{H_{\mathrm{ev}}a_{\mathrm{ev}}^2}{H_{\mathrm{d}}a_{\mathrm{eq}}^2}\right)^{1/2}=\left.\frac{\rho_\phi}{\rho_{AD}}\right|_{\mathrm{ev}}\left(\frac{H_{\mathrm{ev}}}{H_{\mathrm{d}}}\right)^{1/2}. \tag{58}$$

[1]One may wonder if this assumption is inappropriate because the baryon asymmetry created in this channel would be washed out by anomalous electroweak processes [22]. As will be seen later, however, we can avoid this difficulty because in most cases of our interest the AD condensate evaporates at a lower temperature when these anomalous processes are no longer effective.

Thus for $\kappa=1$ the dilution factor is only 14, while for $\kappa=0.1$ it is 70.

Finally, we find that the baryon asymmetry after dilaton decay is given by

$$\frac{n_b}{s}=\frac{n_b}{s}\bigg|_{\rm ev}\frac{1}{\Delta}\simeq 0.1-0.001. \tag{59}$$

Thus in this model the dilution of the asymmetry originally produced from the decay of the AD field is not sufficient.

B. Affleck-Dine mechanism with nonrenormalizable potential

As we have seen, additional entropy production due to the dilaton decay is too small to dilute the baryon asymmetry generated in the original Affleck-Dine scenario. Therefore it is necessary to suppress the generated baryon asymmetry. The presence of nonrenormalizable terms can reduce the expectation value of the AD field during inflation. As a result, the magnitude of the baryon asymmetry can be suppressed. Hence we introduce the following nonrenormalizable term in the superpotential to lift the Affleck-Dine flat direction as a cure to regulate the baryon asymmetry [23,24]:

$$W=\frac{\lambda}{nM^{n-3}}\phi_{AD}^n, \tag{60}$$

where M is some large mass scale.

The potential for the AD field in the inflaton dominated stage reads

$$V(\phi_{AD})=-c_1H^2|\phi_{AD}|^2+\left(\frac{c_2\lambda H\phi_{AD}^n}{nM^{n-3}}+\text{H.c.}\right)+|\lambda|^2\frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}}, \tag{61}$$

where c_1 and c_2 are constants of order unity. The first and second terms are soft terms which arise from the supersymmetry breaking effect due to the vacuum energy of the inflaton. The minimum of the potential is reached at

$$|\phi_{AD}|\simeq\left(\frac{HM^{n-3}}{\lambda}\right)^{1/(n-2)}. \tag{62}$$

During inflation, the AD field takes the expectation value $|\phi_{AD}|\simeq(H_{\rm inf}M^{n-3}/\lambda)^{1/(n-2)}$, where $H_{\rm inf}$ is the Hubble parameter during inflation. After inflation, it also traces the instantaneous minimum, Eq. (62), until the potential is modified and the field becomes unstable there. The AD field starts oscillation when its effective mass becomes larger than the Hubble parameter. There are three possible contributions to trigger this oscillation: the low-energy supersymmetry breaking terms, the thermal mass term from the one-loop effect [25], and the thermal effect at two-loop order [26].

First we consider the case that the low-energy supersymmetry breaking terms are most important and that these thermal effects are negligible. When $H\sim m_{3/2}$, the low-energy supersymmetry breaking terms appear and the potential for the AD field becomes

$$V(\phi_{AD})=m_{AD}^2|\phi_{AD}|^2+\left(\frac{Am_{3/2}\phi_{AD}^n}{nM^{n-3}}+\text{H.c.}\right)+|\lambda|^2\frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}}, \tag{63}$$

where A is a constant of order unity, and the ratio of the energy density of the AD field ρ_{AD} to that of the inflaton ρ_I is given by

$$\frac{\rho_{AD}}{\rho_I}\simeq\left(\frac{m_{3/2}M^{n-3}}{\lambda}\right)^{2/(n-2)}, \tag{64}$$

up to numerical coefficients depending on c_1, c_2, and A. For example, the typical value for $n=4$ is $\rho_{AD}/\rho_I\simeq 10^{-16}(M/\lambda)$.

Let us first assume that the inflaton decayed at $H=m_{AD}$ ($\simeq m_{3/2}$) with the decay rate $\Gamma_I=m_{AD}$ and after that the universe was dominated by radiation. Note that the corresponding reheating temperature is $T_R\simeq\sqrt{\Gamma_I}\simeq\sqrt{m_{AD}}\simeq 10^{10}$ GeV. Then the evaporation of the AD condensate into relativistic plasma would be different from the evaporation into cold plasma considered in the previous subsection. Due to the interaction with the plasma the products of the evaporation acquire a much larger temperature than in the case of evaporation into vacuum. Since the energy density of the condensate is negligible in comparison with the total energy density of the plasma, the temperature of the latter drops in the usual way, $T\propto 1/a$, in contrast to the previously considered case when $T=$const. Since the temperature of the plasma is high, $T\gg m_{AD}$, the baryon number density is $n_b=B_cT^3+C_BT^2\mu$ where $C_B\sim 1$ is a constant coefficient, $\mu\leqslant m_{AD}$ is the value of the chemical potential, and we have neglected terms of the order of μ^3. Since $n_b\propto a^{-3}$, the ratio of $a_{\rm ev}$ to a_{AD} is

$$\frac{a_{\rm ev}}{a_{AD}}=\frac{n_b|_{H=m_{AD}}}{m_{AD}T_R^2} \tag{65}$$

$$\simeq 10\left(\frac{m_{AD}}{1\ \text{TeV}}\right)\left(\frac{10^{10}\ \text{GeV}}{T_R}\right)^2\left(\frac{M}{\lambda}\right)\quad\text{for}\quad n=4 \tag{66}$$

[compare to Eq. (52)]. Here we took for the initial value of the baryonic charge density $n_b|_{H=m_{AD}}=\kappa m_{AD}\phi_{AD}^2$ with $\kappa\sim 1$. For $n=4$, $a_{\rm ev}/a_{AD}$ becomes of order 10, and we find that the condensate will evaporate soon.

To be more precise, however, we must take into account that the interaction rate of the condensate is $\Gamma_{AD}=(0.1$–$0.01)m_{AD}$ and the evaporation cannot start before $H=\Gamma_{AD}$. At that moment the plasma temperature is smaller by the factor $(m_{AD}/\Gamma_{AD})^2=10^2$–$10^4$ and the baryon number density of the condensate is smaller by $(m_{AD}/\Gamma_{AD})^6$. Correspondingly, the redshift of the end of evaporation should be shifted by a factor $(m_{AD}/\Gamma_{AD})^2$ with respect to the beginning of evaporation, and this means that it will remain the same with respect to the initial moment $H=m_{AD}$.

As we have already noted, for $n=4$ the condensate decays quickly and the baryon number density produced in the decay is diluted by the plasma created by the inflaton decay as

$$\frac{n_b}{s}\simeq\frac{T_R}{m_{AD}}\frac{\rho_{AD}}{\rho_I}=10^{-9}\left(\frac{T_R}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{1\ \mathrm{TeV}}{m_{AD}}\right)\left(\frac{M}{\lambda}\right), \tag{67}$$

where T_R is the reheating temperature of the inflaton and we used the estimate of Eq. (64). The result does not depend upon the moment of decay of the AD condensate, since its energy density remains subdominant. If an additional dilution by the dilaton and early oscillation by the thermal effect [25] are operative, the baryon asymmetry become even smaller than the observed one and the $n=4$ model cannot explain the observed baryon asymmetry. Hence we must consider the flat direction with $n>4$.

Hereafter, we study AD fields with $n>4$ including the thermal effect. For AD fields with $n>4$, the relevant thermal effect comes from the running of the gauge coupling constant [26] rather than the thermal plasma effect [25]. The potential for the AD field in the inflaton dominated stage reads

$$V(\phi_{AD})=(-c_1H^2+m_{AD}^2)|\phi_{AD}|^2+\alpha T^4\ln\left(\frac{|\phi_{AD}|^2}{T^2}\right)+\left(\frac{c_2\lambda H\phi_{AD}^n}{nM^{n-3}}+\frac{Am_{3/2}\phi_{AD}^n}{nM^{n-3}}+\mathrm{H.c.}\right)+|\lambda|^2\frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}}, \tag{68}$$

where the second term is the thermal effect at two-loop order, which is pointed out in [26], and α denotes the gauge coupling.

If the effective mass of the AD field becomes comparable to the Hubble parameter when it is larger than the low-energy supersymmetry breaking scale,

$$\alpha\frac{T^4}{|\phi_{AD}|^2}\simeq H^2\quad(>m_{AD}^2), \tag{69}$$

then the AD field undergoes early oscillation by the thermal effect. During the stage dominated by the oscillating inflaton $(t<t_{\mathrm{rh}})$, the temperature of the plasma behaves as

$$T\simeq T_R\left(\frac{a(t_{\mathrm{rh}})}{a(t)}\right)^{3/8}\simeq T_R^{1/2}H^{1/4}. \tag{70}$$

From Eqs. (62) and (70), the effective mass term of the AD field is rewritten as

$$\alpha\frac{T^4}{|\phi_{AD}|^2}\simeq\alpha T_R^2\left(\frac{\lambda}{M^{n-3}}\right)^{2/(n-2)}H^{(n-4)/(n-2)}. \tag{71}$$

Comparing it with Eq. (69), when the AD field begins to oscillate at $t\equiv t_{\mathrm{os}}$, we find that the Hubble parameter is given by

$$H_{\mathrm{os}}\simeq(\alpha T_R^2)^{(n-2)/n}\left(\frac{\lambda}{M^{n-3}}\right)^{2/n}. \tag{72}$$

From now on we concentrate on the case $n=6$, for which

$$H_{\mathrm{os}}\simeq 1\ \mathrm{TeV}\left(\frac{\alpha}{10^{-2}}\right)^{2/3}\left(\frac{T_R}{10^8\ \mathrm{GeV}}\right)^{4/3}\left(\frac{\lambda}{M^3}\right)^{1/3}. \tag{73}$$

Thus we find that the AD field begins to oscillate at $H\gtrsim m_{AD}$ due to the thermal term if $T_R\gtrsim10^8$ GeV for $M=1$ and if $T_R\gtrsim10^6$ GeV for $M=10^{-2}$. During the early oscillation driven by the thermal term, the amplitude of the AD field decreases as

$$|\phi_{AD}(t)|=|\phi_{AD}|_{t_{\mathrm{os}}}\left(\frac{a_{\mathrm{os}}}{a(t)}\right)^{9/4}, \tag{74}$$

where a_{os} denotes the scale factor at the beginning of the oscillation. The analytical derivation of Eq. (74) is shown in the Appendix, and we have confirmed this result by numerical calculation. It also agrees with the analysis in [27]. Using Eqs. (62), (70), and (74), we obtain the ratio of the amplitude of the AD field to the temperature of the plasma as

$$\frac{|\phi_{AD}|}{T}=\left(\frac{M^3}{T_R^2\lambda}\right)^{1/4}\left(\frac{a_{\mathrm{os}}}{a(t)}\right)^{15/8}. \tag{75}$$

We consider the evaporation rate of the AD field. The condition that the particles coupled to the AD field are light enough to exist to the same extent as radiation reads $h|\phi_{AD}|<T$, where h is the corresponding coupling constant. Following [25] let us adopt the scattering rate of the AD field $\Gamma\sim h^4T$ in this situation as the rate of its evaporation. Estimating $h|\phi_{AD}|/T$ and Γ at the reheating time, we find

$$\left.\frac{h|\phi_{AD}|}{T}\right|_{t_{\mathrm{rh}}}\simeq10^{-1}\left(\frac{T_R}{10^{16}\ \mathrm{GeV}}\right)^{1/3}\left(\frac{h}{10^{-2}}\right)\left(\frac{10^{-2}}{\alpha}\right)^{5/6}\left(\frac{M^3}{\lambda}\right)^{2/3} \tag{76}$$

and

$$\frac{\Gamma}{H}\simeq\frac{h^4}{T_R}\simeq\left(\frac{10^{10}\ \mathrm{GeV}}{T_R}\right)\left(\frac{h}{10^{-2}}\right)^4. \tag{77}$$

Equation (76) shows that the particles coupled to the AD field are thermally excited and populated well before the reheating time for any reheating temperature $T_R\lesssim10^{16}$ GeV, while we find that the AD condensate can evaporate around the typical reheating time from Eq. (77).

The baryon number density for the AD field ϕ_{AD} is given by

$$n_b = -iq(\phi_{AD}^* \dot{\phi}_{AD} - \dot{\phi}_{AD}^* \phi_{AD}), \tag{78}$$

where q is the baryonic charge for the AD field.

The baryon number density at $H = H_{os}$ is estimated as

$$n_b|_{t_{os}} = \frac{4qm_{3/2}}{3HM^3} \mathrm{Im}(A\phi_{AD}^6)\bigg|_{t_{os}} = \frac{4q\,\delta m_{3/2}}{3\lambda}\left(\frac{H_{os}M^3}{\lambda}\right)^{1/2}, \tag{79}$$

where δ is the effective relative CP phase. The baryon-to-entropy ratio at the reheating time is estimated as

$$\frac{n_b}{s}\bigg|_{t_{rh}} = \frac{3T_R}{4}\frac{n_b}{\rho_I}\bigg|_{t_{os}} = \frac{q\,\delta m_{3/2}}{3\lambda}\frac{T_R}{H_{os}^2}\left(\frac{H_{os}M^3}{\lambda}\right)^{1/2}. \tag{80}$$

Since the dilution factor by dilaton decay is given by

$$\Delta = \frac{T_R}{10^4 T_D}\left(\frac{\rho_\phi/\rho_I|_{t_{min}}}{10^{-4}}\right), \tag{81}$$

from Eqs. (80) and (81) we obtain the final baryon asymmetry

$$\frac{n_b}{s} = \frac{n_b}{s}\bigg|_{t_{rh}}\frac{1}{\Delta}$$
$$\simeq 10^{-8} q\,\delta\left(\frac{m_{3/2}}{H_{os}}\right)^{3/2}\left(\frac{M}{\lambda}\right)^{3/2}\left(\frac{T_D}{10^{-1}\ \mathrm{GeV}}\right)$$
$$\times\left(\frac{1\ \mathrm{TeV}}{m_{3/2}}\right)^{1/2}\left(\frac{\rho_\phi/\rho_I|_{t_{min}}}{10^{-4}}\right)^{-1} \tag{82}$$

for $H_{os} > m_{3/2}$. This result can easily meet the observations if we take, for example, $H_{os} \simeq 10^2 m_{3/2}$ and other factors to be of the order of unity. In this case, from Eq. (77), we find that for the present case the AD condensate can evaporate before the inflaton decay is completed.

On the other hand, in the case where the reheating temperature is so low that the thermal effect does not lead to early oscillation, the reduction of the cutoff scale M can lead to reasonable baryon asymmetry

$$\frac{n_b}{s} \simeq 10^{-11} q\,\delta\left(\frac{M}{10^{-2}\lambda}\right)^{3/2}\left(\frac{T_D}{10^{-1}\ \mathrm{GeV}}\right)$$
$$\times\left(\frac{1\ \mathrm{TeV}}{m_{3/2}}\right)^{1/2}\left(\frac{\rho_\phi/\rho_I|_{t_{min}}}{10^{-4}}\right)^{-1}. \tag{83}$$

This expression implies that the cutoff scale M should be around the grand unified theory (GUT) scale of 10^{-2} or 10^{16} GeV for inflation models with a low reheating temperature $T_R \lesssim 10^6$ GeV. Furthermore, we find that the final baryon asymmetry is independent of the reheating temperature of inflation within this range.

In the present model, the supersymmetry breaking is caused by the F term of the dilaton. Therefore, when the dilaton decays, it can decay into gravitinos through their mass term, and this process could lead to overproduction of gravitinos. The constraint derived in [28] to avoid the overproduction is $m_\phi \gtrsim 100$ TeV. The mass of the dilaton, Eq. (32), in the model considered here is in the allowed region.

V. CONCLUSION

In this paper, we have studied Affleck-Dine baryogenesis in the framework of string cosmology. In string models, the dilaton is ubiquitous and does not have any potential perturbatively. We adopted a nonperturbatively induced potential of the dilaton via gaugino condensation in the hidden gauge sector. Then we set phenomenologically desired values for the gravitino mass and the VEV of the dilaton.

An attractive mechanism to stabilize the dilaton at the desired minimum was proposed by Barreiro *et al.* [7]. They did not estimate the energy density of the oscillating dilaton. It is estimated in the presented paper, where we have found $\rho_\phi \simeq 10^{-4}\rho$ at $H = m_\phi$. This energy transforms into radiation after the decay of the dilaton before nucleosynthesis because the mass $m_\phi \simeq 10^2$ TeV is sufficiently high.

We have discussed cosmological baryogenesis in this model. In the above-mentioned cosmological history with entropy production, Affleck-Dine baryogenesis might be the only workable mechanism for baryogenesis. We have investigated the Affleck-Dine baryogenesis with and without nonrenormalizable terms. We have shown that, while the original Affleck-Dine scenario produces too much baryon asymmetry even if there is the dilution by the dilaton decay, the model with $n = 6$ nonrenormalizable terms can lead to an appropriate baryon asymmetry.

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APPENDIX

In this appendix, we derive Eq. (74). Although a similar discussion can be found in the literature [29], we derive it for completeness.

We consider the evolution of the AD field after the beginning of oscillation induced by the thermal effect at the two-loop level. Then the AD field obeys the equation of motion

$$\ddot{\phi}_{AD} + 3H\dot{\phi}_{AD} + \alpha\frac{T^4}{\phi_{AD}^*} = 0. \tag{A1}$$

By decomposing ϕ_{AD} into

$$\phi_{AD} = |\phi_{AD}|e^{i\theta} \equiv \Phi e^{i\theta}, \tag{A2}$$

Eq. (A1) is reduced to the following equations:

$$\ddot{\Phi}+3H\dot{\Phi}-\dot{\theta}^2\Phi+\alpha\frac{T^4}{\Phi}=0, \tag{A3}$$

$$(a^3\dot{\theta}\Phi^2)^{\cdot}=0. \tag{A4}$$

The second equation (A4) is interpreted as the conservation of the angular momentum, which corresponds to the baryon number density, and is rewritten as

$$\dot{\theta}\Phi^2=\dot{\theta}\Phi^2|_{t_{\rm os}}\left(\frac{a_{\rm os}}{a(t)}\right)^3\equiv m\Phi_0^2\left(\frac{a_{\rm os}}{a(t)}\right)^3, \tag{A5}$$

where Φ_0 represents the initial amplitude of the AD field and m means the initial angular velocity of the order of $m_{3/2}$ for $n=6$. By eliminating $\dot{\theta}$ in Eqs. (A3) and (A4), we obtain

$$\ddot{\Phi}+3H\dot{\Phi}-m^2\left(\frac{a_{\rm os}}{a(t)}\right)^6\left(\frac{\Phi_0}{\Phi}\right)^4\Phi+\alpha\frac{T^4}{\Phi}=0. \tag{A6}$$

Multiplied by $\dot{\Phi}$ and using $T^4\propto a^{-3/2}$, Eq. (A6) yields

$$\frac{d}{dt}\left[\dot{\Phi}^2+m^2\left(\frac{a_{\rm os}}{a(t)}\right)^6\frac{\Phi_0^4}{\Phi^2}+\alpha T^4\ln\frac{\Phi^2}{T^2}\right]=-6H\left[\dot{\Phi}^2+m^2\left(\frac{a_{\rm os}}{a(t)}\right)^6\frac{\Phi_0^4}{\Phi^2}\right]-\frac{3}{2}H\alpha T^4\ln\frac{\Phi^2}{T^2}+\frac{3}{4}H\alpha T^4. \tag{A7}$$

On the other hand, multiplying Eq. (A6) by Φ, we obtain

$$\frac{1}{a^3}(\Phi a^3\dot{\Phi})^{\cdot}-\dot{\Phi}^2-m^2\left(\frac{a_{\rm os}}{a(t)}\right)^6\frac{\Phi_0^4}{\Phi^2}+\alpha T^4=0. \tag{A8}$$

By taking the time average over the time scale of the cosmic expansion, we obtain the cosmic virial theorem

$$\left\langle\dot{\Phi}^2+m^2\left(\frac{a_{\rm os}}{a(t)}\right)^6\frac{\Phi_0^4}{\Phi^2}\right\rangle=\langle\alpha T^4\rangle, \tag{A9}$$

where $\langle\cdots\rangle$ denotes the time average. Moreover, since the second term, which represents the centrifugal force, becomes efficient only around $\Phi\approx 0$, Eq. (A9) can be rewritten as

$$\langle\dot{\Phi}^2\rangle=\langle\alpha T^4\rangle. \tag{A10}$$

From Eqs. (A7) and (A10), we obtain

$$\frac{d}{dt}\left(1+\ln\frac{\Phi^2}{T^2}\right)=-\frac{15}{4}H. \tag{A11}$$

Since we know $T\propto a^{-3/8}$, we find

$$\Phi\propto a^{-9/4} \tag{A12}$$

for $\Phi\gtrsim T$.

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